Resoluções

Capítulo 16

Inequações modulares

ATIVIDADES PARA SALA

01 a) $x - 3 \geq 7$ ou $x - 3 \leq -7$
$x \geq 10$ $x \leq -4$
$S = \{x \in \mathbb{R} \mid x \leq -4 \text{ ou } x \geq 10\}$

b) $|x - 9| < 6$
$-6 < x - 9 < 6$
$3 < x < 15$
$S = \{x \in \mathbb{R} \mid 3 < x < 15\}$

02 $5 - |x - 3| \geq 0 \Rightarrow |x - 3| \leq 5$
$-5 \leq x - 3 \leq 5$
$-2 \leq x \leq 8$

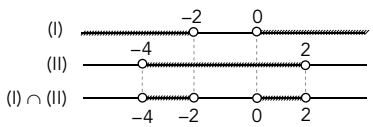
$D = \{x \in \mathbb{R} \mid -2 \leq x \leq 8\}$

03 **E**
$|2x - 1| < 3$
$-3 < 2x - 1 < 3$
$-2 < 2x < 4$
$-1 < x < 2$

04 a) $\overbrace{1 < \underbrace{|x + 1|}_{(II)} < 3}^{(I)}$

(I) $1 < |x + 1|$
$x + 1 > 1$ ou $x + 1 < -1$
$x > 0$ $x < -2$

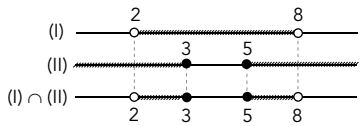
(II) $|x + 1| < 3$
$-3 < x + 1 < 3$
$-4 < x < 2$



$S = \{x \in \mathbb{R} \mid -4 < x < -2 \text{ ou } 0 < x < 2\}$

b) $|x^2 - 1| > 2$
$x^2 - 1 > 2$ ou $x^2 - 1 < -2$
$x^2 > 3$ $x^2 < -1$ (absurdo para $x \in \mathbb{R}$)
$x < -\sqrt{3}$ ou $x > \sqrt{3}$
$S = \{x \in \mathbb{R} \mid x < -\sqrt{3} \text{ ou } x > \sqrt{3}\}$

05 **D**
$|x - 5| < 3$ $|x - 4| \geq 1$
$-3 < x - 5 < 3$ $x - 4 \geq 1$ ou $x - 4 \leq -1$
$2 < x < 8$ (I) $x \geq 5$ $x \leq 3$ (II)



Soma dos inteiros: 3 + 5 + 6 + 7 = 21

ATIVIDADES PROPOSTAS

01 $|x - 2| - 3 \neq 0$
$|x - 2| \neq 3 \Rightarrow x - 2 \neq 3$ ou $x - 2 \neq -3$
$x \neq 5$ $x \neq -1$

$D = \{x \in \mathbb{R} \mid x \neq -1 \text{ ou } x \neq 5\}$

02 $|2x^2 - 1| < 1$

$\overbrace{-1 < \underbrace{2x^2 - 1}_{(II)} < 1}^{(I)}$

(I) $2x^2 - 1 > -1$
$2x^2 > 0$
$x^2 > 0 \Rightarrow x \neq 0$

(II) $2x^2 - 1 < 1$
$2x^2 < 2$
$x^2 < 1 \Rightarrow$ 

$S = (I) \cup (II) = \{x \in \mathbb{R} \mid -1 < x < 1 \text{ e } x \neq 0\}$

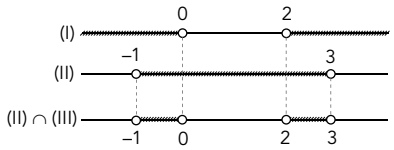
03 $|x^2 - 9x + 9| - 9 \geq 0$
$|x^2 - 9x + 9| \geq 9$
$x^2 - 9x + 9 \geq 9$ ou $x^2 - 9x + 9 \leq -9$
$x^2 - 9x \geq 0$ $x^2 - 9x + 18 \leq 0$
$x \leq 0$ ou $x \geq 9$ (I) $3 \leq x \leq 6$ (II)

$S = (I) \cup (II) = \{x \in \mathbb{R} \mid x \leq 0 \text{ ou } 3 \leq x \leq 6 \text{ ou } x \geq 9\}$

04 $\overbrace{1 < \underbrace{|x - 1|}_{(II)} < 2}^{(I)}$

(I) $1 < |x - 1|$
$x - 1 > 1$ ou $x - 1 < -1$
$x > 2$ $x < 0$

(II) $|x - 1| < 2$
$-2 < x - 1 < 2$
$-1 < x < 3$



$S = \{x \in \mathbb{R} \mid -1 < x < 0 \text{ ou } 2 < x < 3\}$

05 **D**

$$\underbrace{1 < \overbrace{|x-2|}^{\text{(II)}} < 2}_{\text{(I)}}$$

(I) $|x-2| > 1$

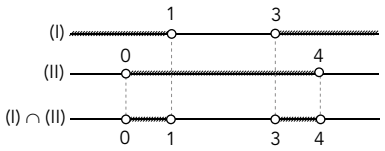
$x - 2 > 1$ ou $x - 2 < -1$

$x > 3$ $\quad$ $x < 1$

(II) $|x-2| < 2$

$-2 < x - 2 < 2$

$0 < x < 4$



06 **C**

$$\underbrace{3 < \overbrace{|2x-1|}^{\text{(I)}} \leq 5}_{\text{(II)}}$$

(I) $3 < |2x-1|$

$2x - 1 > 3$

$x > 2$

ou

$2x - 1 < -3$

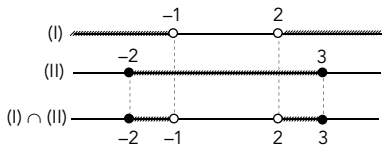
$x < -1$

(II) $|2x-1| \leq 5$

$-5 \leq 2x - 1 \leq 5$

$-4 \leq 2x \leq 6$

$-2 \leq x \leq 3$



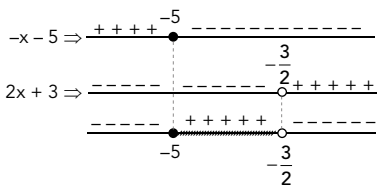
$S = \{x \in \mathbb{R} \mid -2 \leq x < -1 \text{ ou } 2 < x \leq 3\}$

07 (I) $\dfrac{x-2}{2x+3} \geq 1$

$$\frac{x-2}{2x+3} - 1 \geq 0$$

$$\frac{x-2-2x-3}{2x+3} \geq 0$$

$$\frac{-x-5}{2x+3} \geq 0$$



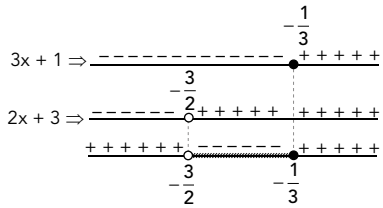
ou

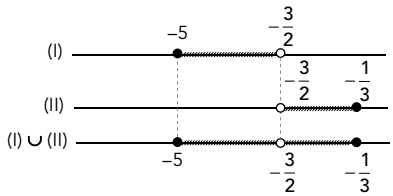
(II) $\dfrac{x-2}{2x+3} \leq -1$

$$\frac{x-2}{2x+3} + 1 \leq 0$$

$$\frac{x-2+2x+3}{2x+3} \leq 0$$

$$\frac{3x+1}{2x+3} \leq 0$$





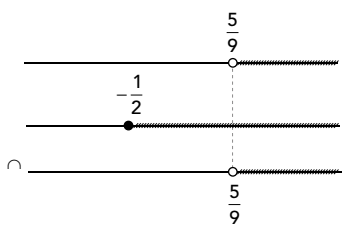
$$S = \left\{x \in \mathbb{R} \mid -5 \leq x < -\frac{3}{2} \text{ ou } -\frac{3}{2} < x \leq -\frac{1}{3}\right\}$$

08 $|4x + 2| > -5x + 7$

$$\underbrace{|4x+2|}_{f(x)} + \underbrace{5x-7}_{g(x)} > 0$$

	$-\frac{1}{2}$	
$f(x)$	$-4x - 2$	$4x + 2$
$g(x)$	$5x - 7$	$5x - 7$
$f(x) + g(x)$	$x - 9$	$9x - 5$

- $x - 9 > 0 \Rightarrow x > 9$, se $x \leq -\frac{1}{2}$ (absurdo)
- $9x - 5 = 0 \Rightarrow x > \frac{5}{9}$, se $x \geq -\frac{1}{2}$



$$S = \left\{x \in \mathbb{R} \mid x > \frac{5}{9}\right\}$$

09 **C**

(Cálculo de S) $x^2 - 3x + 2 = 0$
$x = 1$ ou $x = 2$

(Cálculo de T) $|x - 1| < 3$
$-3 < x - 1 < 3$
$-2 < x < 4$

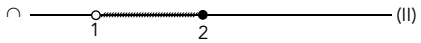
$T = \{-1, 0, 1, 2, 3\}$
$S = \{1, 2\}$
$T - S = \{-1, 0, 3\}$

10 $|x^2 - 4| < 3x$

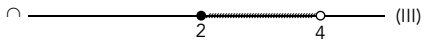
$\underbrace{|x^2 - 4|}_{f(x)} - \underbrace{3x}_{g(x)} < 0$

		-2	2
$f(x)$	$x^2 - 4$	$-x^2 + 4$	$x^2 - 4$
$g(x)$	$3x$	$3x$	$3x$
$f(x) - g(x)$	$x^2 - 3x - 4$	$-x^2 - 3x + 4$	$x^2 - 3x - 4$

(I) $x^2 - 3x - 4 < 0 \Rightarrow -1 < x < 4$, se $x \leq -2$ (absurdo)
(II) $-x^2 - 3x + 4 < 0 \Rightarrow x < -4$ ou $x > 1$, se $-2 \leq x \leq 2$



(III) $x^2 - 3x - 4 < 0 \Rightarrow -1 < x < 4$, se $x \geq 2$



$S = (I) \cup (II) \cup (III) = \{x \in \mathbb{R} \mid 1 < x < 4\}$