Módulo 7 — Trigonometria VII – Função tangente e funções inversas

Atividades para sala

01 **C**

Para Porto Alegre:

$T(30°) = 12 + 3,31 \text{ tg } 30° = 12 + 3,31 \cdot \frac{\sqrt{3}}{3} \cong 12 + 3,31 \cdot \frac{1,73}{3}$

$\cong 13,9$ horas.

Para Macapá (latitude zero):

$T(0°) = 12 + 3,31 \cdot \text{tg } 0° = 12$ horas

Assim, Porto Alegre tem 1,9 hora de sol a mais que Macapá, ou seja: 1,9 hora = 1,9 · 60 minutos = 114 minutos = 1 hora e 54 minutos.

02 **D**

Seja a função $f(x) = \text{tg } x$, então:

I. $D(f) = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi\right\}$.

II. $\text{Im}(f) = \mathbb{R}$.

III. $P = \frac{\pi}{|k|}$, em que **k** é a constante que multiplica **x** na função.

Considere a função $f(x) = 4 \cdot \text{tg}\left(\frac{x}{3} - \frac{\pi}{2}\right)$.

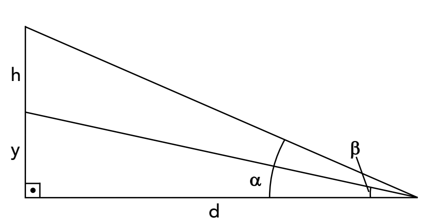
Período:

$P = \frac{\pi}{|k|} \Rightarrow P = \frac{\pi}{\frac{1}{3}} \Rightarrow P = 3\pi$

Domínio:

$\frac{x}{3} - \frac{\pi}{2} \neq \frac{\pi}{2} + k\pi \Rightarrow \frac{x}{3} \neq \pi + k\pi \Rightarrow x \neq 3\pi + 3k\pi$

03 **A**



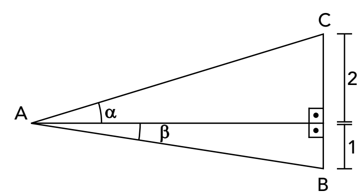
$\text{tg } \alpha = \frac{y}{d} \Rightarrow y = d \cdot \text{tg } \alpha$ (I)

$\text{tg } \beta = \frac{y + h}{d} \Rightarrow h = d \cdot \text{tg } \beta - y$ (II)

Logo, substituindo (I) em (II):

$h = d \cdot \text{tg } \beta - d \cdot \text{tg } \alpha$, então, $h = d\,(\text{tg } \beta - \text{tg } \alpha)$

04 **D**



$$\text{tg}(\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \cdot \text{tg } \beta} \Rightarrow \text{tg}(\alpha + \beta) = \frac{\frac{2}{10} + \frac{1}{10}}{1 - \frac{2}{10} \cdot \frac{1}{10}} \Rightarrow$$

$$\text{tg}(\alpha + \beta) = \frac{\frac{3}{10}}{1 - \frac{2}{100}} = \frac{\frac{3}{10}}{\frac{98}{100}} = \frac{3}{10} \cdot \frac{100}{98} \Rightarrow$$

$$\text{tg}(\alpha + \beta) = \frac{15}{49} \Rightarrow \text{tg } B\hat{A}C = \frac{15}{49} \Rightarrow \text{arc tg}\left(\frac{15}{49}\right) = B\hat{A}C$$

05 **A**

$$\cos\left(\text{arc tg } \frac{\text{energia reativa}}{\text{energia ativa}}\right) = FP > 0,92$$

- **Indústria A:**

  $\text{arc tg } \frac{1\,200}{1\,200} = \alpha \Rightarrow \text{arc tg } 1 = \alpha \Rightarrow \text{tg } \alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$

  Logo, $\cos \alpha = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \cos \alpha \cong 0,70 < 0,92$.

- **Indústria B:**

  $\text{arc tg } \frac{1\,000}{1\,732} = \alpha \Rightarrow \text{tg } \alpha = \frac{1\,000}{1\,732} = \frac{1}{\frac{1\,732}{1\,000}} \Rightarrow$

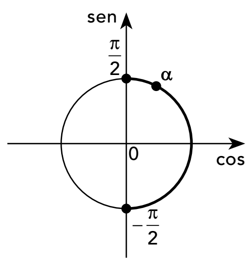
  $\text{tg } \alpha \cong \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \alpha = \frac{\pi}{6}$

  Logo, $\cos \alpha = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \Rightarrow \cos \alpha \cong 0,86 < 0,92$.

06 **A**

Considerando a expressão $E = \cos\left(\text{arc sen}\,\frac{3}{5}\right)$ e fazendo $\alpha = \text{arc sen}\,\frac{3}{5}$, tem-se:

I. $\text{sen}\,\alpha = \frac{3}{5}$ e $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.



II. Como sen α é positivo, **α** pertence ao primeiro quadrante e E = cos α é positivo.

III. $\text{sen}^2\,\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \frac{9}{25} \Rightarrow$

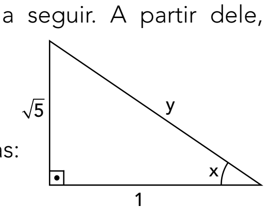
$\cos\alpha = \pm\sqrt{\frac{25-9}{25}} \Rightarrow \cos\alpha = \pm\frac{4}{5}$.

Portanto, $E = \cos\alpha = \frac{4}{5}$, pois **α** é do 1º quadrante.

Atividades propostas

01 **E**

Observe o triângulo retângulo a seguir. A partir dele, deduz-se:



$\text{tg}\,x = \frac{\sqrt{5}}{1}$

Aplicando o Teorema de Pitágoras:

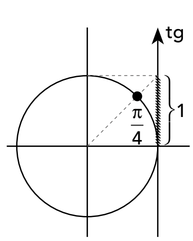
$y^2 = 5 + 1 \Rightarrow y = \sqrt{6}$

Logo: $\text{sen}\,x = \pm\frac{\sqrt{5}}{y} = \pm\frac{\sqrt{5}}{\sqrt{6}} = \pm\frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$\text{sen}\,x = \pm\frac{\sqrt{30}}{6} \Rightarrow \text{sen}^2\,x = \frac{30}{36} = \frac{5}{6}$

02 **B**

Como **x** pertence ao 1º quadrante, observa-se:



Portanto, existe uma única raiz: $x = \frac{\pi}{4}$

03 **B**

$f(x) = \text{tg}\left(3x - \frac{\pi}{2}\right)$

$x = \frac{\pi}{2} \Rightarrow f\left(\frac{\pi}{2}\right) = \text{tg}\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) = \text{tg}\,\pi = 0$

$x = \frac{\pi}{4} \Rightarrow f\left(\frac{\pi}{4}\right) = \text{tg}\left(\frac{3\pi}{4} - \frac{\pi}{2}\right) = \text{tg}\,\frac{\pi}{4} = 1$

Dessa forma, $f\left(\frac{\pi}{2}\right) - f\left(\frac{\pi}{4}\right) = 0 - 1 = -1$.

04 **A**

$h = \left[\cos\left(\frac{7\pi}{6}\right) - 3 \cdot \text{sen}\left(\frac{\pi}{6} + \pi\right)\right] \cdot \text{tg}\left(\frac{5\pi}{4}\right) \Rightarrow$

$h = \left[-\cos\left(\frac{\pi}{6}\right) - 3 \cdot \text{sen}\left(-\frac{\pi}{6}\right)\right] \cdot \text{tg}\left(\frac{\pi}{4}\right) \Rightarrow$

$h = \left[-\frac{\sqrt{3}}{2} + 3 \cdot \frac{1}{2}\right] \cdot (1) \Rightarrow h = \left[-\frac{\sqrt{3}}{2} + \frac{3}{2}\right] \cdot (1) \Rightarrow$

$h = \left[\frac{3 - \sqrt{3}}{2}\right]$

05 **C**

$h(x) = \frac{\sqrt{3}}{3} \Rightarrow \text{tg}\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{3} \Rightarrow \text{tg}\left(2x - \frac{\pi}{4}\right) = \text{tg}\,\frac{\pi}{6} \Rightarrow$

$2x - \frac{\pi}{4} = \frac{\pi}{6} + k\pi \Rightarrow 2x = \frac{\pi}{4} + \frac{\pi}{6} + k\pi \Rightarrow 2x = \frac{5\pi}{12} + k\pi \Rightarrow$

$x = \frac{5\pi}{24} + \frac{k\pi}{2}$

Dessa forma:

- Se $k = 0 \Rightarrow x = \frac{5\pi}{24}$;
- Se $k = 1 \Rightarrow x = \frac{5\pi}{24} + \frac{\pi}{2} \Rightarrow x = \frac{17\pi}{24}$;
- Se $k = 2 \Rightarrow x = \frac{5\pi}{24} + \pi \notin [0, \pi]$.

Logo, $P = \left\{\frac{5\pi}{24}, \frac{17\pi}{24}\right\} \Rightarrow$ 2 elementos.

06 **D**

Para a resolução dessa questão, podem ser utilizadas as seguintes relações, já estudadas em aulas anteriores.

- $\text{tg}\,(a + b) = \frac{\text{tg}\,a + \text{tg}\,b}{1 - \text{tg}\,a \cdot \text{tg}\,b}$
- $\text{tg}\,(a - b) = \frac{\text{tg}\,a - \text{tg}\,b}{1 + \text{tg}\,a \cdot \text{tg}\,b}$

Dessa forma:

$$\text{tg}(x-y) = \frac{\text{tg } x - \text{tg } y}{1+\text{tg } x \cdot \text{tg } y} = 5 - 2y - 2x \text{ (I)}$$

$$\text{tg}(y-x) = \frac{\text{tg } y - \text{tg } x}{1+\text{tg } y \cdot \text{tg } x} = 7 - x - y$$

(–1)

$$\frac{\text{tg } x - \text{tg } y}{1+\text{tg } y \cdot \text{tg } x} = -7 + x + y \text{ (II)}$$

Fazendo (I) = (II):

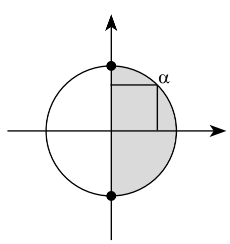
$$5 - 2y - 2x = -7 + x + y$$

$$12 = 3x + 3y \underset{\div 3}{\Rightarrow} x + y = 4$$

07 **D**

$$\text{tg}\Big[\underbrace{\text{arc sen}\frac{2\sqrt{2}}{3}}_{\alpha}\Big] = ? \Rightarrow \text{arc sen}\frac{2\sqrt{2}}{3} = \alpha \Rightarrow \frac{2\sqrt{2}}{3} = \text{sen } \alpha.$$

Note que sen $\alpha > 0$, logo, cos $\alpha > 0$.



$$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$$

Porém, $\text{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \text{sen}^2\alpha \Rightarrow$

$$\cos^2\alpha = 1 - \left(\frac{2\sqrt{2}}{3}\right)^2 \Rightarrow \cos^2\alpha = 1 - \frac{4\cdot 2}{9} \Rightarrow$$

$$\cos^2\alpha = \frac{1}{9} \xrightarrow{\cos\alpha > 0} \cos\alpha = \frac{1}{3}.$$

Dessa forma, $\text{tg}\,\alpha = \frac{\text{sen}\,\alpha}{\cos\alpha} \Rightarrow \text{tg}\,\alpha = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} \Rightarrow \text{tg}\,\alpha = 2\sqrt{2}.$

08 **A**

$$\underbrace{\text{arc tg}\frac{1}{2}}_{x} + \underbrace{\text{arc tg}\frac{1}{3}}_{y} = x + y?$$

$$\text{arc tg}\frac{1}{2} = x \Rightarrow \text{tg } x = \frac{1}{2}$$

$$\text{arc tg}\frac{1}{3} = y \Rightarrow \text{tg } y = \frac{1}{3}$$

$$\text{tg}(x+y) = \frac{\text{tg } x + \text{tg } y}{1 - \text{tg } x \cdot \text{tg } y} \Rightarrow \text{tg}(x+y) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} \Rightarrow$$

$$\Rightarrow \text{tg}(x+y) = 1 \Rightarrow x + y = \frac{\pi}{4}$$

09 **B**

$$\text{Seja } E = \cos\Big[\underbrace{\text{arc sen}\left(\frac{3}{5}\right)}_{\alpha} + \underbrace{\text{arc cos}\left(\frac{4}{5}\right)}_{\beta}\Big].$$

$$\text{arc sen}\frac{3}{5} = \alpha \Rightarrow \frac{3}{5} = \text{sen}\,\alpha \therefore \cos\alpha = \frac{4}{5}.$$

$$\text{arc cos}\frac{4}{5} = \beta \Rightarrow \frac{4}{5}\cos\beta \therefore \text{sen}\,\beta = \frac{3}{5}.$$

Portanto, $\alpha = \beta$.

Dessa forma: $E = \cos(\alpha + \beta) \Rightarrow$

$E = \cos\alpha \cdot \cos\beta - \text{sen}\,\alpha \cdot \text{sen}\,\beta$

$$E = \frac{4}{5}\cdot\frac{4}{5} - \frac{3}{5}\cdot\frac{3}{5} = \frac{7}{25}$$

10 **B**

$$\underbrace{\text{arc sen } x}_{a} + \underbrace{\text{arc sen } 2x}_{b} = \frac{\pi}{2} \Rightarrow a + b = \frac{\pi}{2}$$

$\text{arc sen } x = a \Rightarrow \text{sen } a = x$

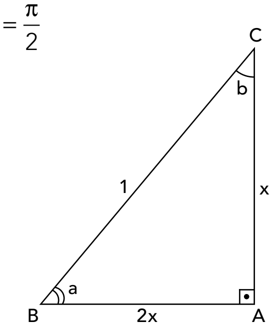
$\text{arc sen } 2x = b \Rightarrow \text{sen } b = 2x$

Portanto:

$x^2 + (2x)^2 = 1 \Rightarrow$

$x^2 + 4x^2 = 1 \Rightarrow$

$$5x^2 = 1 \Rightarrow x = \frac{\sqrt{5}}{5}$$



11 **D**

$$\begin{cases} \cos^2 p - 2\cdot\text{sen } q = 0 \text{ (I)} \\ \cos^2 p + 2\cdot\text{sen } q = \frac{3}{2} \text{ (II)} \end{cases}$$

De (II) – (I), obtém-se:

$$4\text{ sen } q = \frac{3}{2} \Rightarrow$$

$$\text{sen } q = \frac{3}{8} \Rightarrow$$

$$q = \text{arc sen}\frac{3}{8}$$

12 **C**

$$\text{arc tg} = \frac{\sqrt{3}}{3} = \alpha \Rightarrow \text{tg}\,\alpha = \frac{\sqrt{3}}{3} \Rightarrow \alpha = 30°$$

$$\text{arc sen}\frac{\sqrt{3}}{2} = \beta \Rightarrow \text{sen}\,\beta = \frac{\sqrt{3}}{2} \Rightarrow \beta = 60°$$

Logo:

$$\text{tg}\left(5\text{ arc tg}\frac{\sqrt{3}}{2} - \frac{1}{4}\text{arc sen}\frac{\sqrt{3}}{2}\right) = \text{tg}\left(5\cdot 30° - \frac{1}{4}\cdot 60°\right) =$$

$$\text{tg}(150° - 15°) = \text{tg}\,135° = -\text{tg}\,45° = -1$$